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Einstein gravity on a brane in 5D non-compact flat spacetime — DGP model revisited

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Abstract: We revisit the 5D gravity model by Dvali, Gabadadze, and Porrati (DGP). Within their framework it was shown that even in 5D non-compact Minkowski space (x^μ, z), the newtonian gravity can emerge on a brane at short distances by introducing a brane-localized 4D Einstein-Hilbert term $\delta(z)M_4^2\sqrt{|\bar{g}_4|}\bar{R}_4$ in the action. Based on this idea, we construct simple setups in which graviton standing waves can arise, and we introduce brane-localized z derivative terms as a correction to $\delta(z)M_4^2\sqrt{|\bar{g}_4|}\bar{R}_4$. We show that the gravity potential of brane matter becomes $-1/r$ at *long* distances, because the brane-localized z derivative terms allow only a smooth graviton wave function near the brane. Since the bulk gravity coupling may be arbitrarily small, strongly interacting modes from the 5D graviton do not appear. We note that the brane metric utilized to construct $\delta(z)M_4^2\sqrt{|\bar{g}_4|}\bar{R}_4$ can be relatively different from the bulk metric by a conformal factor, and show that the graviton tensor structure that the 4D Einstein gravity predicts are reproduced in the DGP type models.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Classical Theories of Gravity.

Since Kaluza and Klein proposed the five dimensional (5D) theories, it had been believed for a long time that an extra space, if it exists, should be compactified on an extremely small manifold. The newtonian gravity theory, which explains well the observed gravity interactions, seemingly ensures that our space should be effectively three dimensional. As noted in refs. [1, 2, 3], however, the size of the extra dimension(s) could be as large as $(\mathrm{TeV})^{-1}$ scale [1, 2], and may even be infinite provided the graviton is effectively localized on a four dimensional (4D) sub-space (brane) embedded in a 5D AdS spacetime [3].

Especially in ref. [4], Dvali, Gabadadze, and Porrati (DGP) argued that the newtonian gravity can be compatible even with *5D non-compact flat* spacetime, only if (i) the relevant matter fields are localized on a 4D brane, and (ii) a 4D Einstein-Hilbert term $M_4^2\bar{R}_4$ is additionally introduced on the brane apart from the bulk gravity kinetic term $M_5^3R_5$. In ref. [4], it was claimed that the ordinary newtonian potential arises at short distances, whereas at long distances the potential becomes that of a 5D theory. Thus, M_5 should be supposed to be extremely small ($\ll$ TeV) so that the 4D gravity potential is modified at longer distances than the Hubble length scale. This setup was employed in the self-tuning model of the cosmological constant [5].

As shown in ref. [4], however, the graviton tensor structure in the 5D (minimal) DGP model is given by that in tensor-scalar gravity theory rather than that in the Einstein theory.[1] Thus, an extra scalar polarization degree is also involved in 4D gravity interaction. This gives rise to unacceptable deviation from the observation results on light bending around the sun[2] as in the massive gravity case [7].[3] Moreover, in ref. [10] the authors criticized the DGP model pointing that extremely small M_5 possibly induces strong gravity interactions by $h_{\mu 5}$ and h_{55} (whose kinetic terms were supposed to be provided only from $M_5^3R_5$ in the paper). Hence, the validity of momentum expansion would break down with extremely small M_5.

In this paper, we revisit the DGP model with more considerable ingredients, and discuss the long distance gravity potential and the graviton tensor structure again. We consider non-compact 5D spacetime (x^μ, z) $(\mu = 0,1,2,3)$ with the Z_2 symmetry, under which z and $-z$ are identified. We assign even (odd) parity of Z_2 to $g_{\mu\nu}$ and g_{55} $(g_{\mu 5})$. Since 5D general covariance is explicitly broken at the Z_2 fixed point (brane), we require only 4D general covariance on the brane [4]. Let us consider the following action,

$$S = \int d^4x dz \left[\sqrt{|g_5|}\left(\frac{M_5^3}{2}R_5 + \mathcal{L}_m^B\right) + \delta(z)\sqrt{|\bar{g}_4|}\left(\frac{M_4^2}{2}\bar{R}_4 + \mathcal{L}_m^b\right)\right], \tag{1}$$

where $g_5 \equiv \mathrm{Det}g_{MN}$ $(M,N = 0,1,2,3,5)$, $\bar{g}_4 \equiv \mathrm{Det}g_{\mu\nu}$. $\mathcal{L}_m^b$ $(\mathcal{L}_m^B)$ denotes brane (bulk) matter contributions to the action. In this paper, we regard all the standard model fields as brane matter fields. In eq. (1), we dropped the bulk cosmological constant and the

[1]In ref. [6], it is demonstrated that in $D \geq 6$ the brane-localized gravity kinetic term exactly gives the result of the Einstein gravity on the 4D brane.

[2]To compensate the additional attractive force by the extra scalar mode, the authors in ref. [4] suggested to introduce a vector field, which is universally coupled to all matter fields with an U(1) charge.

[3]In ref. [8], it was argued that the resummation of nonlinear effects in massive gravity recovers the result of the Einstein gravity near the sun. This issue in DGP setup is handled in refs. [9].

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brane tension. We assume that they somehow vanish [5]. While R_5 is the 5D Ricci scalar $g^{MN}R^P{}_{MPN}$, $\bar{R}_4$ is defined as the 4D Ricci scalar $\bar{g}^{\mu\nu}\bar{R}^\rho{}_{\mu\rho\nu}$ $(\mu,\nu,\rho=0,1,2,3)$. Even if $\bar{R}_4$ was not contained in the bare action, it could be radiatively generated below the conformal symmetry breaking scale [4]. Generically the metric $\bar{g}_{\mu\nu}$ defining $\bar{R}_4$ can be relatively different from the bulk metric g_{MN} constructing R_5 by a scale factor $\omega^2(x,z)$,

$$\bar{g}_{\mu\nu}(x,z)\equiv\delta_\mu^M\delta_\nu^N g_{MN}(x,z)\times\omega^2(x,z)\,, \tag{2}$$

which can not be removed by redefining the metric, and its degree should appear in the bulk and/or on the brane. R_5 and $\bar{R}_4$ constructed with g_{MN} and $\bar{g}_{\mu\nu}$ still respect 5D and 4D general covariance in the bulk and on the brane, respectively. Since $\bar{R}_4$ will turn out to be dominant in gravity interaction on the brane, it is more convenient to redefine the metric in such a way that ω^2 appears only in the bulk side for proper interpretation of gravity interactions on the brane.

With vanishing bulk cosmological constant and brane tension, the background metric should be flat, $\bar{g}_{\mu\nu}=\eta_{\mu\nu}$ and ω^2 a constant, which can be normalized to unity by rescaling M_5. Beyond the leading term, however, ω^2 would appear as a non-trivial physical degree in the bulk. The perturbed metric near the flat background is

$$ds^2=\left[1+\frac{1}{2}\phi(x,z)\right]^{-2}\Big(\eta_{\mu\nu}+h_{\mu\nu}(x,z)\Big)dx^\mu dx^\nu+2h_{\mu5}dx^\mu dz+\Big(1+h_{55}(x,z)\Big)dz^2\,, \tag{3}$$

where $\eta_{\mu\nu}\equiv\mathrm{diag}(-1,1,1,1)$, and ϕ indicates the sub-leading term of ω, i.e. $\omega^{-2}\approx(1+\frac{1}{2}\phi)^{-2}\approx(1-\phi)$. On the other hand, on the brane the perturbed metric is just given by $\eta_{\mu\nu}+h_{\mu\nu}$.

The localized gravity kinetic term $\delta(z)M_4^2\bar{R}_4$ in eq. (1) adds a brane-localized 4D Einstein tensor to the 5D full gravity equation [4]. At the linearized level, which is relevant in low energy gravity interactions, it takes the form:

$$G^{(0)}_{\mu\nu}=-\frac{\delta(z)}{2M_5^3}M_4^2\left[\nabla_4^2\bar{h}_{\mu\nu}+\eta_{\mu\nu}\partial^\lambda\partial^\delta\bar{h}_{\lambda\delta}-\partial_\mu\partial^\lambda\bar{h}_{\lambda\nu}-\partial_\nu\partial^\lambda\bar{h}_{\lambda\mu}\right], \tag{4}$$

where ∇_4^2 denotes $\eta^{\mu\nu}\partial_\mu\partial_\nu$, and we defined $\bar{h}_{\mu\nu}\equiv h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h$, $h\equiv\eta^{\mu\nu}h_{\mu\nu}$. The subscripts and superscripts are raised and lowered with $\eta_{\mu\nu}$. As is well known, the linearized Einstein tensor eq. (4) is invariant under the gauge transformation, $h_{\mu\nu}(x,z)\longrightarrow h_{\mu\nu}(x,z)+\partial_\mu\xi_\nu(x,z)+\partial_\nu\xi_\mu(x,z)$. Hence, it would be reasonable to consider also the following brane-localized higher derivative terms as a correction to eq. (4),

$$G^{(1)}_{\mu\nu}=-\alpha\frac{\delta(z)}{2M_5^3}\partial_z^2\left[\nabla_4^2\bar{h}_{\mu\nu}+\eta_{\mu\nu}\partial^\lambda\partial^\delta\bar{h}_{\lambda\delta}-\partial_\mu\partial^\lambda\bar{h}_{\lambda\nu}-\partial_\nu\partial^\lambda\bar{h}_{\lambda\mu}\right], \tag{5}$$

where α is a dimensionless coupling and $\partial_z^2\equiv\partial_z\partial_z$, because eq. (5) still maintains the gauge symmetry and the Z_2 symmetry. Small brane excitation effects would appear as the correction by such z derivative terms. We note that the linearized tensor $G^{(0)}_{\mu\nu}+G^{(1)}_{\mu\nu}$ can be effectively obtained by redefining $h_{\mu\nu}$ in eq. (4) only on the brane

$$h_{\mu\nu}\longrightarrow H_{\mu\nu}=h_{\mu\nu}+\frac{\alpha}{M_4^2}\partial_z^2h_{\mu\nu}\,. \tag{6}$$

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Since eq. (5) respects the gauge symmetry observed in eq. (4), one could expect that it is somehow generated in higher energy scales. To get eqs. (4) and (5) in the equation of motion, let us consider the following brane-localized gravity kinetic and interaction terms in the linearized lagrangian,

$$\mathcal{L}_{\rm lin} = -\delta(z)\left[\frac{M_4^2}{4}\left(\frac{1}{2}(\partial_\mu H_{\nu\rho})^2 - \frac{1}{2}(\partial_\mu H)^2 - (\partial^\nu H_{\mu\nu})^2 + \partial_\mu H \partial_\nu H^{\mu\nu}\right) - \frac{1}{2}H_{\mu\nu}T^{\mu\nu}\right], \quad (7)$$

where $H \equiv \eta^{\mu\nu}H_{\mu\nu}$ and $T_{\mu\nu}(x)$ indicates the energy-momentum tensor by brane-localized matter fields. Hence, unlike in the bulk metric, the perturbed metric on the brane is effectively given by $H_{\mu\nu}$. By embedding $\eta_{\mu\nu} + H_{\mu\nu}$ in the modified brane "metric" $\hat{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{\alpha}{M_4^2}\partial_z^2\bar{g}_{\mu\nu}$, a generally covariant 4D lagrangian can be constructed, $\delta(z)\sqrt{|\bar{g}_4|}\ \bar{R}_4[\bar{g}_{\mu\nu}] \longrightarrow \delta(z)\sqrt{|\hat{g}_4|}\ \hat{R}_4[\hat{g}_{\mu\nu}]$. A 4D general coordinate transformation still can be defined as $\partial_z^2\bar{g}'_{\mu\nu} = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}\partial_z^2\bar{g}_{\rho\sigma}$ with $\partial_z^2(\frac{\partial x^\rho}{\partial x'^\mu})|_{z=0} = 0$. The variation $\delta\partial_z^2 h_{\mu\nu}|_{z=0}$ of eq. (7), which is independent of $\delta h_{\mu\nu}|_{z=0}$, leads to a constraint equation,

$$G_{\mu\nu}^{(0)} + G_{\mu\nu}^{(1)} = \frac{1}{M_5^3}\delta(z)T_{\mu\nu}(x)\,. \quad (8)$$

Indeed, the variation $\delta\partial_z^2 h_{\mu\nu}|_{z=0}$ can not be converted to $\delta h_{\mu\nu}|_{z=0}$ through a partial integration, because the partial integration for $\partial_z^2 h_{\mu\nu}|_{z=0}$ on the brane induces physically ill-defined functions such as $\partial_z^2\delta(z)$ and $\partial_z\delta(z)$ [11]. The extremizing condition for $\mathcal{L}_{\rm lin}$ under $\delta h_{\mu\nu}$ yields the same expression, but it takes part in the 5D gravity equation,

$$G_{\mu\nu}^B + G_{\mu\nu}^{(0)} + G_{\mu\nu}^{(1)} = \frac{1}{M_5^3}\delta(z)T_{\mu\nu}(x) + \frac{1}{M_5^3}T_{\mu\nu}^B(x,z)\,, \quad (9)$$

where $G_{\mu\nu}^B$ and $T_{\mu\nu}^B$ are the linearized bulk Einstein tensor and the bulk energy-momentum tensor, respectively. Hence, eq. (8) implies $G_{\mu\nu}^B(x,z) = \frac{1}{M_5^3}T_{\mu\nu}^B(x,z)$.

Once we introduce such brane-localized higher derivative terms shown in eq. (7), in fact, it is perturbatively consistent to consider also other higher order curvature terms like R_5^2, $R_{MN}R^{MN}$, $R_{MNPQ}R^{MNPQ}$, etc. In this paper, in order to see the effect by the terms in eq. (5) clearly in the linearized gravity equation, we assume that the higher order curvature terms in the lagrangian are given by the Gauss-Bonnet type,

$$\sqrt{|g_5|}\beta M_5\left(R_5^2 - 4R_{MN}^2 + R_{MNPQ}^2\right) + \delta(z)\sqrt{|\bar{g}_4|}\gamma\left(\bar{R}_4^2 - 4\bar{R}_{\mu\nu}^2 + \bar{R}_{\mu\nu\rho\sigma}^2\right), \quad (10)$$

which leaves intact the linearized gravity equation derived from eq. (7) with the flat background spacetime [12]. Indeed, in supergravity the quadratic curvature terms would appear as the Gauss-Bonnet type. Moreover, even if supersymmetry is broken on the brane, supersymmetry in the bulk can remain exact if the extra dimension size is infinite [6, 13]. Actually, brane higher curvature terms with a combination different from the Gauss-Bonnet ratio do not seriously change our conclusion.

With the 4D harmonic gauge, $\partial^\mu(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h) = 0$, which fixes a gauge parameter $\xi_\mu(x,z)$, the linearized Einstein equation reads [5, 14]

$$
\begin{aligned}
(\mu\nu):\ & \left[\partial_\mu\partial_\nu\left(\phi - \frac{1}{2}h_{55}\right) - \eta_{\mu\nu}\nabla_4^2\left(\phi - \frac{1}{2}h_{55}\right) - \frac{3}{2}\eta_{\mu\nu}\partial_z^2\phi\right] + \\
& + \frac{1}{2}\left[\partial_\mu\partial^z h_{\nu 5} + \partial_\nu\partial^z h_{\mu 5} - 2\eta_{\mu\nu}\partial^\lambda\partial^z h_{\lambda 5}\right] - \frac{1}{2}\left[\nabla_5^2\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) - \frac{1}{2}\eta_{\mu\nu}\partial_z^2 h\right] - \\
& - \frac{\delta(z)}{2M_5^3}\left[\left(M_4^2 + \alpha\partial_z^2\right)\nabla_4^2\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right)\right] = \frac{1}{M_5^3}\delta(z)T_{\mu\nu}(x)\,, \qquad (11) \\
(\mu 5):\ & -\frac{1}{2}\left[\nabla_4^2 h_{\mu 5} - \partial_\mu\partial^\lambda h_{\lambda 5}\right] + \frac{3}{2}\partial_\mu\partial_z\phi - \frac{1}{4}\partial_\mu\partial_z h = 0\,, \qquad (12) \\
(55):\ & -\frac{3}{2}\nabla_4^2\phi + \frac{1}{4}\nabla_4^2 h = 0\,, \qquad (13)
\end{aligned}
$$

where ∇_5^2 (∇_4^2) indicates $\eta^{\mu\nu}\partial_\mu\partial_\nu + \partial_z^2$ ($\eta^{\mu\nu}\partial_\mu\partial_\nu$). The terms in the third line of eq. (11) came from the brane-localized terms in the action eq. (1). The presence of non-vanishing but small energy-momentum tensor by brane matter $T_{\mu\nu}$ are responsible for metric fluctuation near the flat background. Here we neglected $T^B_{\mu\nu}(x,z)$ for simplicity. The equation from $\delta S/\delta\phi = 0$ for the action S turns out to be just $\eta^{MN}G^B_{MN} = \eta^{MN}T^B_{MN} = 0$, which is consistent with eq. (8).

Eqs. (11) and (12) are invariant under

$$
\begin{aligned}
h_{\mu 5}(x,z) &\longrightarrow h_{\mu 5}(x,z) + \partial_\mu\xi_5(x,z)\,, \qquad (14) \\
h_{55}(x,z) &\longrightarrow h_{55}(x,z) + 2\partial_z\xi_5(x,z)\,. \qquad (15)
\end{aligned}
$$

We can choose $\xi_5(x,z)$ such that

$$h_{55}(x,z) = 2\phi(x,z) \qquad (16)$$

is satisfied. From eqs. (11) and (12), the dynamics of $h_{\mu 5}$ is governed by $\partial_{(\mu}h_{\nu)5} = \eta_{\mu\nu}\partial^\lambda h_{\lambda 5}$ $(= 0)$ and a boundary condition $h_{\mu 5}|_{z=0} = 0$. Eqs. (12) and (13) are easily solved by setting

$$h(x,z) = 6\phi(x,z)\,. \qquad (17)$$

Then, the last terms of the first and second lines in eq. (11) cancel out, and so eq. (11) becomes much simpler

$$\nabla_5^2\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) + \frac{\delta(z)}{M_5^3}\left[\left(M_4^2 + \alpha\partial_z^2\right)\nabla_4^2\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right)\right] = -\frac{2\delta(z)}{M_5^3}T_{\mu\nu}(x)\,. \qquad (18)$$

Hence, the energy-momentum conservation law $\partial^\mu T_{\mu\nu} = 0$ is trivially satisfied. After some algebra, eq. (18) in 4D momentum space (p,z) becomes

$$\left[\left(-p^2 + \partial_z^2\right) - \frac{\delta(z)}{M_5^3}\left(M_4^2 p^2 + \alpha p^2\partial_z^2\right)\right]\tilde{h}_{\mu\nu}(p,z) = -\frac{2\delta(z)}{M_5^3}\left[\tilde{T}_{\mu\nu}(p) - \frac{1}{2}\eta_{\mu\nu}\tilde{T}(p)\right]\,, \qquad (19)$$

where $p^2 \equiv p^\mu p_\mu$ (≤ 0), $\tilde{T} \equiv \eta^{\mu\nu}\tilde{T}_{\mu\nu}$, and tildes indicates 4D Fourier-transformed fields. Note that the tensor structure of $\tilde{h}_{\mu\nu}$ is exactly the same as that in the 4D Einstein gravity theory. Hence, the scalar mode included in $h_{\mu\nu}$ is successively decoupled from low energy gravity interactions between brane matter fields, and only two degrees of freedom in polarization states of the graviton survive as in the 4D Einstein gravity theory. Actually, it was possible by considering ω^{-2} in the bulk metric.

The bulk solution of eq. (19) with the even parity would be given by a linear combination of $\cos kz$ and $\sin k|z|$, where $k \equiv \sqrt{-p^\mu p_\mu}$ (≥ 0). Their coefficients could be determined by the boundary condition at $z=0$. We note that $\partial_z^2(\sin k|z|)$ generates a delta function. Thus, from the last term in the left hand side of eq. (19), $\sin k|z|$ induces a highly singular term proportional to $\delta^2(z)$, which can not be matched to the right hand side of eq. (19) in weakly coupled gravity theory. This singularity can not be removed by introducing a suitable gravity counter term. Hence, the solution satisfying the boundary condition at $z=0$ should be given only by $\cos kz$. It explicitly satisfies also the constraint equation (8) (or $G^B_{\mu\nu}=0$).

A $\cos kz$ type solution, however, implies that an outgoing wave ($e^{ik|z|}$) as well as an incoming wave ($e^{-ik|z|}$) should be generated when brane matter fields are fluctuating. This is inconsistent with causality. A simple way to naturally create incoming wave is to introduce two more branes around the $z=0$ brane. Then the right hand side of eq. (11) is modified into

$$\frac{1}{M_5^3}\delta(z)T_{\mu\nu}(x) + \frac{1}{M_5^3}\Big[\delta(z-z_c)+\delta(z+z_c)\Big]S_{\mu\nu}(x)\,, \tag{20}$$

where the two additional branes are introduced symmetrically under $z \leftrightarrow -z$. This is a $T_{\mu\nu} - h_{\mu\nu} - S_{\mu\nu}$ coupled system. In this setup, the standing waves such as $\cos kz$, $\sin k|z|$ could arise between the $z=\pm z_c$ branes, while still only outgoing wave is allowed in the outside region of the two branes. But the term $\delta(z)\alpha k^2\partial_z^2 h_{\mu\nu}$ in eq. (11) selects only $\cos kz$ type solution at $|z|\leq z_c$. Since low energy matter fluctuations would induce the graviton waves typically with long wave length, the additional branes can be located considerably far from the $z=0$ brane.

The solutions satisfying such boundary conditions are

$$\tilde{h}^I_{\mu\nu}(k,z) = \frac{-2\cos kz}{M_4^2k^2-\alpha k^4}\Big[\tilde{T}_{\mu\nu}(k)-\frac{1}{2}\eta_{\mu\nu}\tilde{T}(k)\Big] \qquad \text{for } |z|\leq z_c\,, \tag{21}$$

$$\tilde{h}^{II}_{\mu\nu}(k,z) = \frac{\cos kz_c\; e^{ik|z|}}{iM_5^3k}\Big[\tilde{S}_{\mu\nu}(k)-\frac{1}{2}\eta_{\mu\nu}\tilde{S}(k)\Big] \qquad \text{for } |z|\geq z_c\,, \tag{22}$$

where $\tilde{S}_{\mu\nu}$ is determined such that the boundary condition at $|z|=z_c$ is fulfilled. $\tilde{S}_{\mu\nu}$ turns out to be related to $\tilde{T}_{\mu\nu}$,

$$\tilde{S}_{\mu\nu}(k) = \frac{-2M_5^3 e^{-i(kz_c-\frac{\pi}{2})}}{M_4^2k-\alpha k^3}\times\tilde{T}_{\mu\nu}(k)\,. \tag{23}$$

Hence, gravity effects at the $z=0$ brane by "dark matter" fluctuations on the $z=\pm z_c$ branes would be very suppressed at low energy. Time evolution of $h_{\mu\nu}(x,z)$ is governed by $\tilde{T}_{\mu\nu}(k)$ ($=\int d^4x e^{ikx}T_{\mu\nu}(x)$).

It is interesting to compare our solutions eqs. (21) and (22) with the solution in the original DGP model,

$$\sim \frac{e^{ik|z|}}{M_4^2 k^2 + 2iM_5^3 k} \,. \tag{24}$$

In the DGP model, typical properties appearing in solutions of 5D theories ($\sim 1/2iM_5^3 k$) and 4D theories ($\sim 1/M_4^2 k^2$) are contained in one solution. At low energies 5D property becomes dominant, while at high energies 4D property appears dominant. On the other hand, in our solution the two properties are separate as shown in eqs. (21) and (22). In view of an observer living in the region $|z| > z_c$, $\tilde{T}_{\mu\nu}$ ($\tilde{S}_{\mu\nu}$) is negligible at low (high) energies. Since the observer at $|z| > z_c$ can not distinguish $\tilde{T}_{\mu\nu}$ and $\tilde{S}_{\mu\nu}$ if the relevant graviton's wave length is long enough $kz_c \ll 1$, the resultant gravity effects are the same as those in the DGP model (upto the tensor structure) to the observer at $|z| > z_c$. However, to an observer living *in* the $z = 0$ brane, the solution describing gravity interaction is always given by eq. (21).

We note that at low energy eq. (21) guarantees the same gravity interaction on the $z = 0$ brane as that in the 4D Einstein gravity theory. M_5 can be arbitrarily large, and so the strongly interacting modes from 5D graviton can be avoided. Only if $k^2 < M_4^2/\alpha$, no ghost particle is excited in eq. (21). With $T_{00}(x) = \rho(x) \gg T_{ii}(x)$ ($i = 1, 2, 3$), the non-relativistic low energy gravity potential on $z = 0$ brane is calculated to be [4]

$$\begin{aligned} V(\vec{r}) &= \int dt \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{1}{2} \tilde{h}_{00}(k, z = 0) \\ &\approx -\frac{1}{8\pi M_4^2} \int d^3\vec{r}' \, \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \,. \end{aligned} \tag{25}$$

The newtonian constant is determined to $G_N \equiv 1/(8\pi M_4^2)$.

In eq. (20), we introduced two matter branes at $z = \pm z_c$ without any localized gravity kinetic terms. In fact, introduction of the z derivative terms at $z = \pm z_c$ is dangerous, because they disallow outgoing waves also outside $z = \pm z_c$ branes. It is unnatural to introduce them only on the $z = 0$ brane. [Thus, it would be more desirable to interpret the interval $|z| \leq z_c$ and $z = \pm z_c$ branes as the inside of a (thick) brane and its surfaces, respectively.]

An alternative way to obtain a standing wave is to introduce bulk matter. Roughly speaking, introduction of bulk matter is nothing but to introduce infinite number of matter branes with making their interval lengths infinitely small. Again this setup is a $T_{\mu\nu} - h_{\mu\nu} - T^B_{\mu\nu}$ coupled system, where $T^B_{\mu\nu}$ denotes the energy-momentum tensor contributed by bulk matter. In the presence of bulk matter, both outgoing and incoming waves are basically possible, and their mixing ratio in a solution would be determined by initial or boundary conditions. For simplicity, let us assume that a bulk matter field is distributed uniformly in the z direction, i.e. $T^B_{MN}(x, z) = T^B_{MN}(x)$, and also assume $T^B_{\mu 5} = 0$. This kind of energy momentum tensor can be provided by a bulk scalar field independent of z. As will be shown below, even when brane matter is absent, the bulk matter uniformly distributed in the z direction compels outgoing and incoming graviton waves to be excited due to the presence of $-\frac{\delta(z)}{M_5^3} M_4^2 p^2 \tilde{h}_{\mu\nu}$ in eq. (19).

The right hand side of eqs. (11) and (13) are modified into

$$(\mu\nu):\ \frac{1}{M_5^3}\delta(z)T_{\mu\nu}(x)+\frac{1}{M_5^3}T^B_{\mu\nu}(x)\,, \tag{26}$$

$$(55):\ \frac{1}{M_5^3}T^B_{55}(x)\,. \tag{27}$$

In that case, eq. (17) should be replaced by

$$\phi(x,z)=\frac{1}{6}\Big[h(x,z)-f(x)\Big]\,, \tag{28}$$

where $f(x)$ satisfies $\frac{1}{4}\nabla_4^2 f(x)=\frac{1}{M_5^3}T^B_{55}(x)$. For the modified equation of motion,

$$\left[\left(k^2+\partial_z^2\right)+\frac{\delta(z)}{M_5^3}\left(M_4^2k^2+\alpha k^2\partial_z^2\right)\right]\tilde{h}_{\mu\nu}(k,z)=-\frac{2}{M_5^3}\left[\delta(z)\tilde{J}_{\mu\nu}(k)+\tilde{J}^B_{\mu\nu}(k)\right], \tag{29}$$

where $\tilde{J}^{(B)}_{\mu\nu}\equiv\tilde{T}^{(B)}_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}\tilde{T}^{(B)}$, we obtain the following solution,

$$\tilde{h}_{\mu\nu}(k,z)=-\frac{2\tilde{J}^B_{\mu\nu}(k)}{M_5^3k^2}-\frac{2\mathrm{cos}kz}{M_4^2k^2-\alpha k^4}\left[\tilde{J}_{\mu\nu}(k)-\frac{M_4^2}{M_5^3}\tilde{J}^B_{\mu\nu}(k)\right]. \tag{30}$$

This solution is consistent with eq. (8) (or $G^B_{\mu\nu}=\frac{1}{M_5^3}T^B(x,z)$). Were it not for the brane-localized kinetic terms, the last two terms with $\mathrm{cos}kz$ would just be a homogeneous part of the solution. We note here that even if $\tilde{J}_{\mu\nu}=0$ but only if $\tilde{J}^B_{\mu\nu}\neq 0$, the graviton solution shows the z dependence. When $\tilde{J}_{\mu\nu}\neq 0$ and $\tilde{J}^B_{\mu\nu}\neq 0$, bulk matter enables the graviton wave to satisfy the boundary conditions by enhancing its incoming part. Since the low energy approximate solution at $z=0$ is

$$\tilde{h}_{\mu\nu}(k,z=0)\approx-\frac{2\tilde{J}_{\mu\nu}(k)}{M_4^2k^2}\left[1+O\left(\frac{\alpha k^2}{M_4^2}\right)\right], \tag{31}$$

the leading term of the graviton solution at low energy coincides with the solution in the 4D Einstein gravity theory.

Although we showed that the 4D Einstein gravity is reproduced on the brane with a special bulk matter field, we arrive at the same conclusion also with a general $T^B_{MN}(x,z)$ by bulk matter. In that case $f(x)$ in eq. (28) should be generalized to $f(x,z)$. For any bulk source $S^B_{\mu\nu}(x,z)$ contributed by $T^B_{\mu\nu}(x,z)$, $\frac{1}{4}\eta_{\mu\nu}\partial_z^2 f(x,z)$, $\frac{1}{2}\partial_\mu\partial^z h_{\nu 5}+\cdots$, and so on, the bulk solution generally has the following form,

$$\tilde{h}_{\mu\nu}(k,z)=-\frac{2}{M_5^3}\int\frac{dk_5}{2\pi}\frac{\mathrm{cos}k_5z}{k^2-k_5^2}\tilde{S}^B_{\mu\nu}(k,k_5)+p_{\mu\nu}(k)\mathrm{cos}kz\,, \tag{32}$$

where $p_{\mu\nu}(k)$ should be determined by the boundary condition at $z=0$. Because of the boundary condition by $\delta(z)M_4^2k^2\tilde{h}_{\mu\nu}$, the solution at $z=0$ reduces to eq. (31) upto $O(\alpha k^2/M_4^2)$ for any arbitrary $S^B_{\mu\nu}(x,z)$.

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In conclusion, we have shown that the 5D DGP type gravitational models can be phenomenologically viable. We introduced two more branes and/or bulk matter to make the graviton's standing waves possible. Apart from bulk and brane gravity kinetic terms $M_5^3\sqrt{|g_5|}R_5$, $\delta(z)M_4^2\sqrt{|\bar{g}_4|}\bar{R}_4$ as in the original DGP model, we consider also the brane-localized z derivative terms at the linearized level in order to allow only smooth graviton waves near the brane. In this model, the long distance gravity potential on the brane turns out to be the newtonian potential. Since M_5 can be arbitrarily large, the strongly interacting modes from the 5D graviton can be avoided. Since the brane metric can be relatively different from the bulk metric by a conformal factor, we can obtain the desired tensor structure of the graviton.

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